The Enumeration Method for Selecting Optimum Switching Network Structures

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***Abstract*— In this paper it is shown that traditional continuous optimization methods for multistage switching networks may fail, leading to non-optimal structures. Then, a simple enumeration method to solve the optimization problem is proposed. The numerical examples show that the parameters of optimum networks can be calculated very fast for any practical network sizes.**

***Index Terms*— Clos network, nonblocking network, switching.**

I. Introduction

OPTIMIZATION of nonblocking multistage switching networks has been one of the hot research issues since publication of the well known paper of Clos [1]. He calculated the optimum parameters of switching networks by using conventional methods of the mathematical analysis, treating all variables of the cost function as continuous ones. In the case where the obtained parameters were not integers or were not realizable, the nearest realizable values were selected. The same approach was used by Pollen for the calculation of tables of optimum nonblocking networks [2]. The method seemed to work in the case when the cost of the network was expressed by the number of crosspoints. The main disadvantage of the continuous method was its complexity, especially for networks containing more than three stages as well as for networks where some constraints were imposed on elementary switch sizes or overall network structure. V. E. Beneš formulated several lemmas and theorems, by using arguments of the number and set theories, giving optimum parameters of rearrangeable networks [3]. It was proven that most of these theorems contained considerable errors, and a corrected version of the theory was proposed in [4]. Later, A. Jajszczyk proved that the continuous optimization method fails if the optimization criterion is the number of elementary switches (or integrated chips), rather than the number of crosspoints [5]. In the same paper, a discrete optimization method, based on the dynamic programming approach, was proposed.

In this paper we will show that the original Clos method, using continuous optimization, also fails in the case where the network cost is expressed by the number of crosspoints. We will also show that a straightforward enumeration method, involving extensive computations, can be efficiently used for optimization of all practical network structures.

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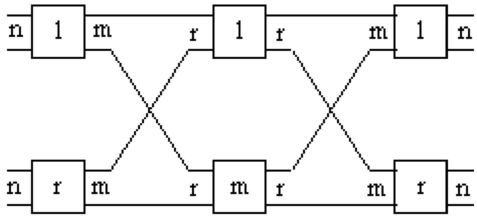

Fig. 1. Three-stage Clos network $\nu(m, n, r)$.

II. Critique of the Continuous Optimization

We will focus here on three- and five-stage strictly-nonblocking Clos networks to illustrate the problem [1]. A three-stage Clos switching network, shown in Fig. 1, can be described by three integers: m, n, r and is denoted as $\nu(m, n, r)$. Such a network is nonblocking in the strict sense if and only if $m \geq 2n - 1$ [1]. A five-stage network can be obtained by replacing middle-stage switches with complete three-stage networks. A similar approach can be used to obtain networks composed of seven, nine, etc. stages.

For $m = 2n - 1$ the total number of crosspoints in such a network is equal to $C(3) = (2n-1)(2N + N^2/n^2)$, where N is the total number of inlets and outlets. As we can see, the number of crosspoints depends only on a single variable (n). For a given value of N, the minimum number of crosspoints for the three stage switching network is achieved when

$$\frac{\mathrm{d}C(3)}{\mathrm{d}n} = 0 \Longleftrightarrow 2n^3 - nN + N = 0 \qquad (1)$$

The above equation can be solved for n, and then both nearest integers should be checked for minima. As mentioned earlier, such an approach can lead to non-optimal solutions. This is illustrated by Table I.

TABLE I

SWITCH SIZES CALCULATED FROM EQ. (1) AND THEIR OPTIMUM VALUES

N	n_{con}	n_{opt}
1000	21.8	20
10000	70.2	69
20000	99.5	100
50000	**157.6**	**152**
100000	**223.1**	**216**
500000	499.5	500

It contains values of the numbers of inputs of first-stage switches calculated from (1), and denoted as n_{con}, for selected N's, and the numbers of inputs for the minimum-cost networks (n_{opt}), calculated by using the method described in Section III. As clearly seen at the marked rows, rounding the results obtained from (1) to the nearest integers leads to selection of non-optimal network structures.

The problem gets even more complex as we approach a greater number of stages, since each produces another variable. For five-stage switching networks the following equation gives the total number of crosspoints:

$$C(5) = (2n_1 - 1)\left[2N + (2n_2 - 1)\left(\frac{2N}{n_1} + \frac{N^2}{n_1^2 n_2^2}\right)\right] \quad (2)$$

where: n_1, n_2 are the numbers of inlets per a first-stage switch, and a second-stage switch, respectively.

To find the minimum of function (2) it is necessary to compare two partial derivatives of $C(5)$ (in respect to n_1 and n_2, respectively) to 0:

$$\begin{cases} \dfrac{\partial C(5)}{\partial n_1} = 0 \\ \\ \dfrac{\partial C(5)}{\partial n_2} = 0 \end{cases} \Longleftrightarrow \begin{cases} n_1 = \dfrac{N(n_2 - 1)}{2n_2^3} \\ \\ N = \dfrac{n_1 n_2^2 (2n_1^2 + 2n_2 - 1)}{(n_1 - 1)(2n_2 - 1)} \end{cases} \quad (3)$$

In the conventional optimization approach, solving (3) and rounding the obtained results to the nearest integers should lead to optimal solutions, similarly as in the three-stage network case. However, as shown in Table II, this is not necessarily true (see marked rows).

TABLE II
SWITCH SIZES CALCULATED FROM EQ. (3) AND THEIR OPTIMUM VALUES

N	$n_{1,con}$	$n_{1,opt}$	$n_{2,con}$	$n_{2,opt}$
1000	11.6	11	6.0	7
10000	**26.1**	**23**	**13.3**	**15**
20000	**33.2**	**31**	**16.83**	**17**
50000	**45.4**	**40**	**22.9**	**25**
100000	57.5	57	29.0	27
500000	99.0	100	49.7	50

III. THE ENUMERATION METHOD

Although some sophisticated, integer optimization methods can be used to find optimum parameters of multistage Clos networks, we applied here a simple enumeration method. The essence of such a method is to enumerate all possible solutions. In other words, to find the optimum solution for given N we check the cost of the network for all possible sizes of elementary switches and select those sizes that minimize the cost. The enumeration methods are now widely used in various research areas, taking advantage of growing computing power. The source codes in C for calculation of optimum three-, five- and seven-stage Clos structures can be found in [6].

The processing time depends on the size of the problem, but also the efficiency of the program code. If the calculation time becomes a significant problem (although not in our case) the programmer may try to optimize the code. For example, we do not have to check all possible network structures, because it is easy to notice, by taking into account properties of the cost function, that some of them cannot be optimal. For example, solving the optimization problem for a three-stage Clos network in a way presented in [6] results in cost calculation for each n from the range of 1 to N. Every time the program calculates the minimum r to achieve the given total number of inlets. Assuming we chose $N = 1000$ and the program has just checked the structure with $n = 50$ and $r = 20$, the next step is to check $n = 51$ and $r = 20$, but since n increased and r did not decrease, the total number of crosspoints has to be greater than in the previous step. Therefore, the structure with $n = 51$ and $r = 20$ cannot be optimal and it can be excluded from direct evaluation.

In the case of switching networks of more than three stages we can use the enumeration procedure recursively. For very large networks the dynamic programming method can be used [5].

Table III contains sample calculation times in seconds for 3-, 5-, and 7-stage strictly nonblocking Clos networks. The program was coded in C and executed on a standard PC with Pentium IV, 1.6 GHz processor, under the Windows operating system.

TABLE III
PROCESSING TIME FOR CALCULATING OPTIMUM STRUCTURES OF 3-, 5-, AND 7-STAGE STRICTLY NONBLOCKING CLOS NETWORKS

N	t_{C3} [s]	t_{C5} [s]	t_{C7} [s]
1000	< 0.01	< 0.01	0.02
10000	0.01	0.03	0.18
100000	0.05	0.42	2.50
1000000	0.54	4.99	32.60

As we can see, even for networks of capacities considerably exceeding practical sizes, the parameters of optimum structures can be calculated within a reasonable time.

IV. CONCLUSION

In this paper we showed that traditional continuous optimization methods for multistage switching networks may fail. We also proposed application of a simple enumeration method to solve the optimization problem. The numerical examples show that the parameters of optimum networks can be calculated very fast for any practical network sizes. Although our examples concerned strictly-nonblocking Clos networks, the same approach can be used also to other network classes, such as repackable or rearrangeable networks. The cost function, we used, was expressed by the number of crosspoints, but other cost criteria (e.g., the number of elementary switches) can also be applied.

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